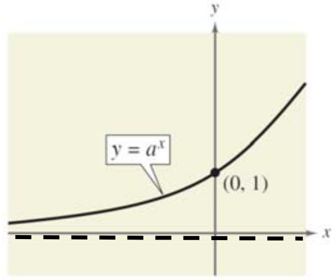
# Chapter 3 Summary Sheet

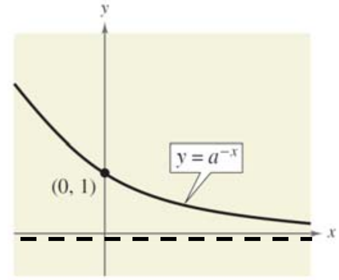
**<u>Exponential Function</u>**: $f(x) = a^x$

### Exponential Growth




*Graph of* $y = a^x, a > 1$
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Intercept: $(0, 1)$
- Increasing
- $x$-axis is a horizontal asymptote ($a^x \to 0$ as $x \to -\infty$)
- Continuous

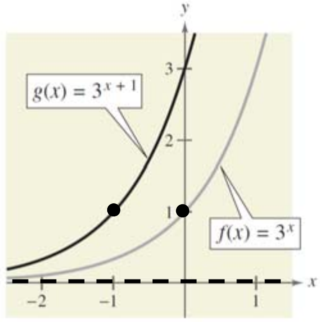
### Exponential Decay



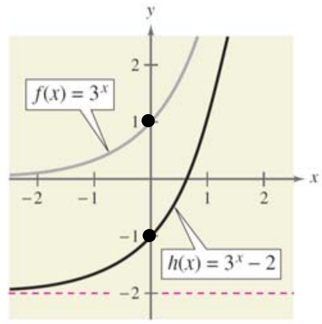

*Graph of* $y = a^{-x}, a > 1$
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- Intercept: $(0, 1)$
- Decreasing
- $x$-axis is a horizontal asymptote ($a^{-x} \to 0$ as $x \to \infty$)
- Continuous

**<u>Exponential Transformations:</u>** **1st:** Shift the starting point and asymptote **2nd:** Apply reflection

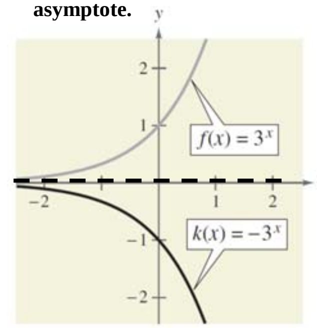
**Horizontal Shift**



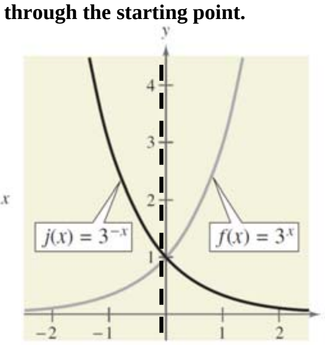

**Vertical Shift**




**Reflect over the horizontal asymptote.**



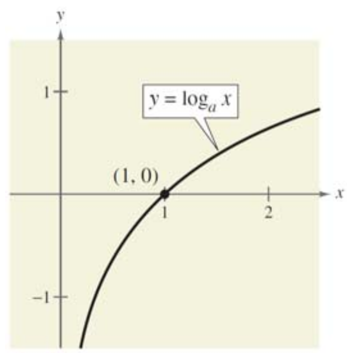

**Reflect over the vertical line passing through the starting point.**




**<u>Logarithmic Function</u>**: $f(x) = \log_a x$ **Important:** The inverse of an exponential function is a logarithmic function.

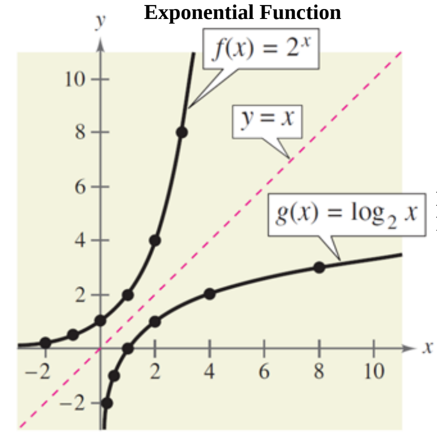
**Recall:** The inverse of a function is reflected over the line y = x.

### Exponential Growth




*Graph of* $y = \log_a x, a > 1$
- Domain: $(0, \infty)$
- Range: $(-\infty, \infty)$
- $x$-intercept: $(1, 0)$
- Increasing
- One-to-one, therefore has an inverse function
- $y$-axis is a vertical asymptote ($\log_a x \to -\infty$ as $x \to 0^+$).
- Continuous
- Reflection of graph of $y = a^x$ about the line $y = x$

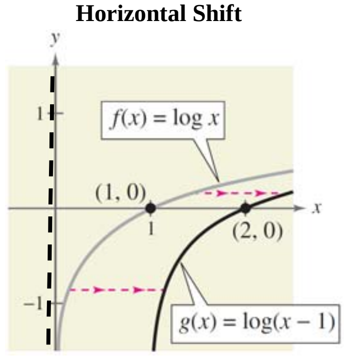
**Exponential Function**



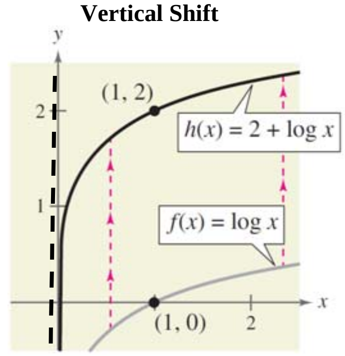

**Logarithmic Function**

**<u>Logarithmic Transformations:</u>** **1st:** Shift the starting point and asymptote **2nd:** Apply reflection

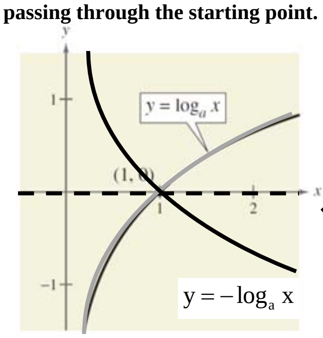
**Horizontal Shift**



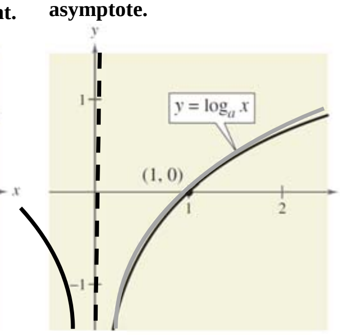

**Vertical Shift**




**Reflect over the horizontal line passing through the starting point.**




**Reflect over the vertical asymptote.**




$y = \log_a(-x)$

**Formulas for Compound Interest**

**Natural Base:** $e \approx 2.718$

**For n compounding per year:** $A = P\left(1+\frac{r}{n}\right)^{nt}$

**For continuous compounding:** $A = Pe^{rt}$

**A:** Account Balance **P:** Principle (Initial Investment) **r:** Interest Rate **n:** Compoundings Per Year **t:** Time in Years

**Logarithmic Form** $y = \log_a x \longleftrightarrow x = a^y$ **Exponential Form**

**Important Notes:**
Bases are the same both in logarithmic and exponential form.

You can NOT take the log of a negative number.

The most common logarithm has base 10. If there is no base for the logarithm, then it is 10.

**Natural Logarithm:** $\ln x = \log_e x$

| **Properties of Logarithms** | **Properties of Natural Logarithms** |
|---|---|
| $\log_a 1 = 0$ because $a^0 = 1$ | $\ln 1 = 0$ because $e^0 = 1$ |
| $\log_a a = 1$ because $a^1 = a$ | $\ln e = 1$ because $e^1 = e$ |
| $\log_a a^x = x$ and $a^{\log_a x} = x$ | $\ln e^x = x$ and $e^{\ln x} = x$ |
| If $\log_a x = \log_a y$, then $x = y$ | If $\ln x = \ln y$, then $x = y$ |

**Change of Base Formula:** $\log_a x = \frac{\log x}{\log a}$ or $\log_a x = \frac{\ln x}{\ln a}$ **Ex:** $\log_5 8 = \frac{\log 8}{\log 5} = \frac{\ln 8}{\ln 5}$

**Properties of Logarithms**

Product Property $\log_a xy = \log_a x + \log_a y$

Quotient Property $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Property $\log_a x^y = y \log_a x$