

Pre-Calculus

Spring Final Review 5

$$\textcircled{1} \quad \sin\left(\frac{3\pi}{2} + \theta\right)$$

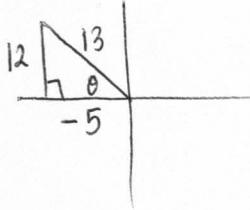
$$= \sin\left(\frac{3\pi}{2}\right) \cos\theta + \cos\left(\frac{3\pi}{2}\right) \sin\theta$$



$$= -\frac{1}{1} \cos\theta + \frac{0}{1} \sin\theta$$

$$= \boxed{-\cos\theta}$$

\textcircled{2}



$$\begin{aligned} \cos 2\theta &= \cos^2\theta + \sin^2\theta \\ &= \left(\frac{-5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 \\ &= \frac{25}{169} - \frac{144}{169} \\ &= \boxed{-\frac{119}{169}} \end{aligned}$$

$$\textcircled{3} \quad \sec\theta \cos\theta + \sin\theta \csc\theta$$

$$\frac{1}{\cos\theta} \cos\theta + \sin\theta \frac{1}{\sin\theta}$$

$$1 + 1 = \boxed{2}$$

$$\textcircled{4} \quad \frac{\cos\theta}{\sec\theta + \tan\theta} \quad (\sec\theta - \tan\theta)$$

$$= \frac{\cos\theta \sec\theta - \cos\theta \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\cos\theta \cdot \frac{1}{\cos\theta} - \cos\theta \frac{\sin\theta}{\cos\theta}}{1}$$

$$= 1 - \sin\theta \quad \checkmark$$

$$\textcircled{5} \quad (\sin\theta + \cos\theta)^2 + 2\sin\theta \cos\theta$$

$$\begin{aligned} &= \sin^2\theta + 2\sin\theta \cos\theta + \cos^2\theta + 2\sin\theta \cos\theta \\ &= \sin^2\theta + \cos^2\theta + 4\sin\theta \cos\theta \end{aligned}$$

$$= 1 + 2(2\sin\theta \cos\theta)$$

$$= \boxed{1 + 2\sin 2\theta}$$

$$\textcircled{6} \quad \frac{1 + \cos 2\theta}{2} = \frac{(1 + \cos^2\theta - \sin^2\theta)}{2}$$

$$= \frac{\cos^2\theta + \cos^2\theta}{2}$$

$$= \cancel{\frac{2\cos^2\theta}{2}}$$

$$= \boxed{\cos^2\theta}$$

$$\textcircled{7} \quad \frac{\sec \theta}{\tan \theta + \cot \theta} = \frac{\sec \theta}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \cdot \cos \theta}$$

$$= \frac{\sec \theta}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} = \frac{\sec \theta}{\frac{1}{\sin \theta \cos \theta}}$$

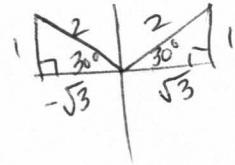
$$= \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta \cos \theta}} \cdot \frac{\sin \theta \cos \theta}{1} = \frac{\sin \theta \cos \theta}{\cos \theta}$$

$= [\sin \theta] \quad \checkmark$

$$\textcircled{9} \quad \cos x - 2 \sin x \cos x = 0$$

$$\cos x (1 - 2 \sin x) = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$



$$\frac{\pi}{2}, \frac{3\pi}{2}$$

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

$$\left| \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \right|$$

$$\textcircled{8} \quad 2 \sin^2 \theta - 11 \sin \theta - 6 = 0$$

Same as $2x^2 - 11x - 6 = 0$

$$\begin{array}{r} \text{Add} \\ -11 \\ \hline -12 \\ \hline -12 \end{array}$$

$$2 \sin^2 \theta - 12 \sin \theta + \sin \theta - 6 = 0$$

Mult

$$2 \sin \theta (\sin \theta - 6) + 1 (\sin \theta - 6) = 0$$

$$(\sin \theta - 6)(2 \sin \theta + 1) = 0$$

$$\begin{array}{r} \text{sin } \theta = \frac{6}{1} \\ \text{Does not exist!} \end{array}$$

$$\begin{array}{r} \text{sin } \theta = -\frac{1}{2} \\ -1 \quad -\frac{\sqrt{3}}{2} \quad \frac{\sqrt{3}}{2} \quad 1 \\ \hline 2 \quad 2 \quad -1 \end{array}$$

$$210^\circ, 330^\circ$$

$$210^\circ \cdot \frac{\pi}{180^\circ} = \frac{7\pi}{6}$$

$$\boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$$

$$11 \cdot 330^\circ \cdot \frac{\pi}{180^\circ} = \frac{11\pi}{6}$$

$$\textcircled{10} \quad 2 \cos^2 \theta + \sin \theta = 2$$

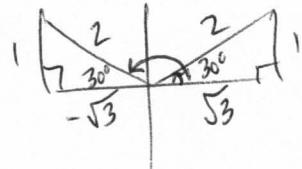
$$2(1 - \sin^2 \theta) + \sin \theta = 2$$

$$2 - 2 \sin^2 \theta + \sin \theta = 2$$

$$-2 \sin^2 \theta + \sin \theta = 0$$

$$\sin \theta (-2 \sin \theta + 1) = 0$$

$$\sin \theta = 0 \quad \sin \theta = \frac{1}{2}$$



$$0, \pi$$

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

$$\boxed{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi}$$

$$\textcircled{11} \quad (x+1)^{\frac{3}{2}} = 64$$

$$\left((x+1)^{\frac{3}{2}} \right)^{\frac{2}{3}} = 64^{\frac{2}{3}}$$

$$x+1 = (\sqrt[3]{64})^2$$

$$x+1 = 4^2$$

$$x+1 = 16 \quad \boxed{x=15} \quad \text{Check } \checkmark$$

$$\textcircled{12} \quad |4x - 12| = 7x + 3$$

$$4x - 12 = 7x + 3$$

$$-3x - 12 = 3$$

$$\cancel{-3x} = 15$$

$$x = -5$$

Check NO!

$$4x - 12 = -(7x + 3)$$

$$4x - 12 = -7x - 3$$

$$11x - 12 = -3$$

$$\cancel{11x} = \frac{9}{11}$$

$$\boxed{x = \frac{9}{11}} \quad \text{Check } \checkmark$$

$$\textcircled{13} \quad g(x) = \begin{cases} x, & \text{if } x < 0 \\ -2, & \text{if } 0 \leq x < 1 \\ x^2, & \text{if } x \geq 1 \end{cases}$$

$$g(x) = x$$

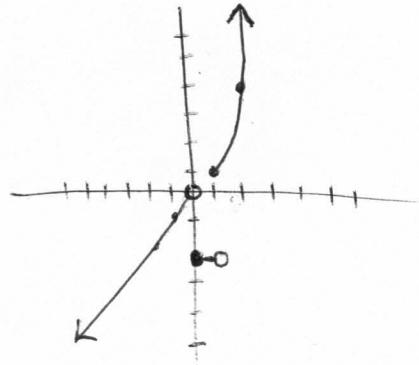
$$\begin{array}{|c|c|} \hline x & y \\ \hline 0 & 0 \\ -1 & -1 \\ -2 & -2 \\ \hline \end{array}$$

$$g(x) = -2$$

$$\begin{array}{|c|c|} \hline x & y \\ \hline 0 & -2 \\ 1 & -2 \\ \hline \end{array}$$

$$g(x) = x^2$$

$$\begin{array}{|c|c|} \hline x & y \\ \hline 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ \hline \end{array}$$



$$\textcircled{14} \quad f(g(x))$$

$$f(x) = 2x + 5 \quad g(x) = x^2 - 10$$

$$f(x^2 - 10) = 2(\cancel{x^2 - 10}) + 5$$

$$= 2x^2 - 20 + 5$$

$$= \boxed{2x^2 - 15}$$

$$\textcircled{15} \quad y = \frac{kx\sqrt{z}}{w}$$

$$3 = \frac{k(2)\sqrt{4}}{16}$$

$$4 \cdot 3 = \frac{4k}{16} \cdot 4 \quad k = 12$$

$$y = \frac{12(15)\sqrt{36}}{8} = 12(3)(6)$$

$$\boxed{y = 216}$$

$$\textcircled{16} \quad \begin{array}{r} 2c^2 - 6c - 9 \\ \hline c^2 - 8c - 8 \end{array} \quad \begin{array}{r} 2c^4 - 6c^3 - 25c^2 + 48c + 72 \\ - 2c^4 + 8c^3 + 16c^2 \end{array}$$

$$\begin{array}{r} -6c^3 - 9c^2 + 48c + 72 \\ + 16c^3 - 8c^2 - 48c \end{array}$$

$$\begin{array}{r} -9c^2 + 8c + 72 \\ + 9c^2 - 8c - 72 \end{array}$$

$$\boxed{0}$$

$$(17) \quad -2 \left| \begin{array}{cccc} 6 & 11 & -4 & -4 \\ \downarrow & -12 & 2 & 4 \\ 6 & -1 & -2 & 0 \end{array} \right.$$

$$x = -2 \rightarrow x + 2 = 0$$

One factor

$$\begin{aligned} 6x^2 - x - 2 &= 6x^2 + 3x - 4x - 2 \\ &= 3x(2x+1) - 2(2x+1) \\ &= (2x+1)(3x-2) \\ 6x^3 + 11x^2 - 4x - 4 &= \boxed{(2x+1)(3x-2)(x+2)} \end{aligned}$$

$$(18) \quad x^3 - x^2 - 3x + 3 = 0$$

$$x^2(x-1) - 3(x-1) = 0$$

$$(x-1)(x^2-3) = 0$$

$$\begin{array}{rcl} x-1=0 & & x^2-3=0 \\ +1 & +1 & +3 & +3 \end{array}$$

$$x = 1 \quad \sqrt{x^2} = \pm\sqrt{3} \quad x = \pm\sqrt{3}$$

$$\boxed{x = 1, x = \pm\sqrt{3}}$$

$$(19) \quad x^4 - 8x^3 - 11x^2 + 18x = 0$$

$$x(x^3 - 8x^2 - 11x + 18) = 0$$

\uparrow

$x = 0$ Factors of Constant $= \frac{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18}{\pm 1}$
 Factors of Leading Coefficient

Test:

$$\begin{array}{r} | 1 & -8 & -11 & 18 \\ | \downarrow & 1 & -7 & -18 \\ 1 & -7 & -18 & 0 \end{array} \quad x = 1$$

$$x^2 - 7x - 18 = 0 \quad 2 \cancel{-9} \\ -18$$

$$(x+2)(x-9) = 0$$

$$\boxed{x = -2, x = 9}$$

$$\boxed{x = 0, x = 1, x = -2, x = 9}$$

$$(20) \quad g(x) = \frac{x^2 + 4x - 12}{x}$$

Vertical Asymptotes: Find zeros of denominator.

$$\boxed{x = 0}$$

Horizontal Asymptote: The degree of the numerator is higher than the denominator. There is no horizontal asymptote.

$$(21) \quad \log_{\frac{1}{3}} 27$$

$$= \log_{\frac{1}{3}} 3^3$$

$$= \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-3}$$

$$= -3 \log_{\frac{1}{3}} \left(\frac{1}{3}\right) = -3(1) = \boxed{-3}$$

$$(22) \log \left(\sqrt[5]{\frac{b^2 c}{d^4}} \right) = \log \left(\frac{b^2 c}{d^4} \right)^{\frac{1}{5}}$$

$$= \frac{1}{5} \log \frac{b^2 c}{d^4}$$

$$= \frac{1}{5} [\log b^2 c - \log d^4]$$

$$= \frac{1}{5} [\log b^2 + \log c - \log d^4]$$

$$= \frac{1}{5} [2 \log b + \log c - 4 \log d]$$

$$= \boxed{\frac{2}{5} \log b + \frac{1}{5} \log c - \frac{4}{5} \log d}$$

$$(23) 9^{1-3x} = 27^{2x-1}$$

$$3^{2(1-3x)} = 3^{3(2x-1)}$$

$$2 \overbrace{(1-3x)} = 3 \overbrace{(2x-1)}$$

$$2 - 6x = 6x - 3$$

$$-2 - 12x = -3$$

$$\cancel{-12} \cancel{x} = \cancel{-5}$$

$$\boxed{x = \frac{5}{12}}$$

$$(24) \log x + \log(x-3) = 1$$

$$\log(x(x-3)) = 1$$

$$x^2 - 3x = 10^1$$

$$\begin{array}{r} -10 \\ -10 \end{array}$$

$$x^2 - 3x - 10 = 0$$

$$\cancel{2} \cancel{-3} \cancel{-5}$$

$$\cancel{-10}$$

$$(x+2)(x-5) = 0$$

$$x = -2 \quad x = 5$$

$$\text{No!} \quad \boxed{x = 5}$$

Check

* Always check
log equation
solution.

$$(25) \log 64 = 2 \log x$$

$$\log 64 = \log x^2$$

$$\sqrt{x^2} = \sqrt{64} \quad x = \pm 8 \quad \text{check}$$

$$\boxed{x = 8}$$

$$(26) y^2 = 5 - x$$

$$x + 5 \cancel{x} = 11$$

$$-8y - 5y \quad x = 11 - 5y$$

$$y^2 = 5 - (11 - 5y)$$

$$y^2 = 5 - 11 + 5y$$

$$y^2 = -6 + 5y$$

$$y^2 - 5y + 6 = 0$$

$$\cancel{-5} \cancel{-3} \quad (y-2)(y-3) = 0$$

$$\cancel{-2} \cancel{6} \quad y=2 \quad y=3$$

$$x + 5(2) = 11$$

$$\cancel{x + 10} = 11$$

$$x = 1$$

$$x + 5(3) = 11$$

$$\cancel{x + 15} = 11$$

$$x = -4$$

$$\boxed{(1, 2) \text{ and } (-4, 3)}$$

$$\begin{array}{l} (27) \quad ① x + 3y + z = 3 \\ ② 2x + 5y - 2z = -4 \\ ③ x + 6y + 2z = 0 \end{array}$$

$$\begin{array}{l} ① \cancel{2x + 6y + 2z = 6} \\ ② \cancel{2x + 5y - 2z = -4} \\ ③ \cancel{x + 6y + 2z = 0} \\ 4x + 11y = 2 \\ 3x + 11y = -4 \end{array}$$

$$\begin{array}{r} -4x - 11y = -2 \\ 3x + 11y = -4 \\ \hline -1x = -6 \\ x = 6 \end{array}$$

$$\begin{array}{r} 3(6) + 11y = -4 \\ 18 + 11y = -4 \\ -18 \qquad -18 \\ 11y = -22 \\ y = -2 \end{array}$$

$$\begin{array}{r} 6 + 3(-2) + z = 3 \\ 6 - 6 + z = 3 \\ z = 3 \\ |(6, -2, 3)| \end{array}$$

$$(28) \quad \frac{7}{2} + \frac{7}{4} + \frac{7}{8} + \dots$$

$$\frac{7}{2} \cdot r = \frac{7}{4} \quad r = \frac{1}{2}$$

$$\frac{7}{4} \cdot r = \frac{7}{8} \quad r = \frac{1}{2}$$

⋮

$$S = \frac{\frac{7}{2}}{1 - \frac{1}{2}} = \frac{\frac{7}{2}}{\frac{1}{2}} = 7$$

7

$$(29) \quad 14 + 11 + 8 + 5 \dots - 82$$

Common Difference: $d = -3$

$$a_n = a_1 + (n-1)d \quad \text{Arithmetic Series}$$

$$-82 = 14 + (n-1)(-3)$$

$$-14 = -14$$

$$-96 = \frac{(n-1)(-3)}{(-3)}$$

$$32 = n - 1 \quad n = 33$$

$$S_{33} = \frac{33}{2} (14 + (-82))$$

$$= \frac{33}{2} (-68) = \boxed{-1,122}$$

$$(30) \quad P(\text{head and heart}) \quad \boxed{\text{Head} \mid \text{Heart}}$$

$$= P(\text{head}) \cdot P(\text{heart})$$

$$= \frac{1}{2} \cdot \frac{13}{52} = \frac{13}{104} = \frac{1}{8}$$

$$\boxed{\frac{1}{8}}$$