

Pre-Calculus Test Chapter 2

Form A

Show ALL work!!!

- 1 Perform the operation and write the result in standard form.

$$(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i)$$

$$= 14 - 10i^2$$

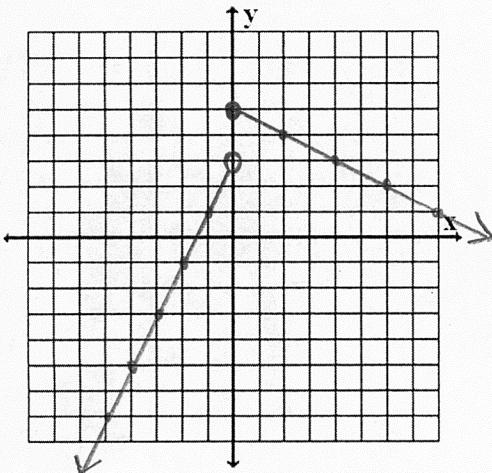
$$= 14 - 10(-1)$$

$$= 14 + 10$$

$$= \boxed{24}$$

- 2 Chapter 1 Graph the piecewise function.

$$f(x) = \begin{cases} 2x + 3, & x < 0 \\ -\frac{1}{2}x + 5, & x \geq 0 \end{cases}$$



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- 3 Find all the zeros of the function, determine the multiplicity of each zero, and make a rough sketch of the graph.

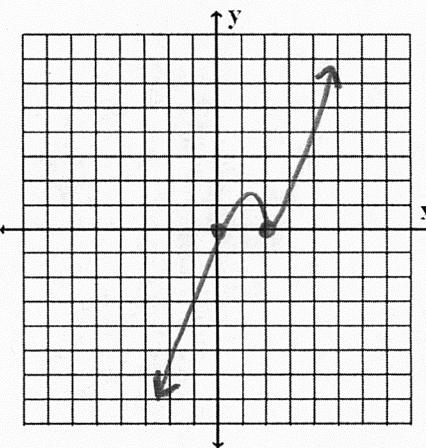
$$f(x) = x^3 - 4x^2 + 4x$$

$$= x(x^2 - 4x + 4)$$

$$\text{Zeros: } x = 0, x = 2$$

$$= x(x-2)(x-2)$$

Multiplicity: 0 has an ODD multiplicity
2 has an EVEN multiplicity



- 4 Chapter 0 Solve the equation and check your solutions.

$$(x-5)^{\frac{3}{2}} = 8$$

$$(9-5)^{\frac{3}{2}} = 8$$

$$\left((x-5)^{\frac{3}{2}}\right)^{\frac{2}{3}} = 8^{\frac{2}{3}}$$

$$4^{\frac{3}{2}} = 8$$

$$x-5 = (\sqrt[3]{8})^2$$

$$\sqrt[2]{4^3} = 8$$

$$x-5 = 2^2$$

$$8 = 8 \checkmark$$

$$\begin{matrix} x-5 = 4 \\ +5 \quad +5 \end{matrix}$$

$$\boxed{x = 9}$$

5 Write the complex number in standard form.

$$4 + \sqrt{-9}$$

$$= 4 + 3i$$

6 State the possible rational zeros of the function.
You don't need to find the zeros.

$$g(x) = x^3 - 4x^2 - x + 4$$

$$\begin{aligned} \text{Possible Rational Zeros} &= \frac{\text{Factors of Constant}}{\text{Factors of Leading Coefficient}} \\ &= \frac{\pm 1, \pm 2, \pm 4}{\pm 1} \end{aligned}$$

$$1, -1, 2, -2, 4, -4$$

7 Use long division to divide.

$$(x^4 + 3x^2 + 1) \div (x^2 - 2x + 3)$$

$$\begin{array}{r} x^2 + 2x + 4 \\ x^2 - 2x + 3 \overline{)x^4 + 0x^3 + 3x^2 + 0x + 1} \\ \underline{-x^4 + 2x^3 - 3x^2} \\ 2x^3 + 0x^2 + 0x + 1 \\ \underline{-2x^3 + 4x^2 - 6x} \\ 4x^2 - 6x + 1 \\ \underline{-4x^2 + 8x - 12} \\ 2x - 11 \end{array}$$

$$x^2 + 2x + 4 + \frac{2x - 11}{x^2 - 2x + 3}$$

8 Find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

$$1, 5i, -5i$$

$$x = 1 \quad x = 5i \quad x = -5i$$

$$x - 1 = 0 \quad x - 5i = 0 \quad x + 5i = 0$$

$$(x - 1)(x - 5i)(x + 5i)$$

$$= (x - 1)(x^2 - 25i^2)$$

$$= (x - 1)(x^2 - 25(-1))$$

$$= (x - 1)(x^2 + 25)$$

$$= x^3 + 25x - x^2 - 25$$

$$= x^3 - x^2 + 25x - 25$$

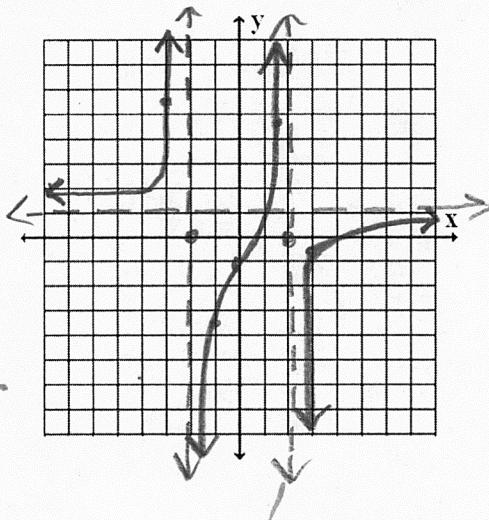
- 9 The axis of symmetry formula can be used to find the vertex of a parabola?

- 10 Graph the function. Use the given information to assist you.

$$h(x) = \frac{x^2 - 5x + 4}{x^2 - 4} = \frac{(x-1)(x-4)}{(x-2)(x+2)}$$

$$h(-3) = 5.6, h(3) = -0.4$$

$$h(-1) = -3.3, h(0) = -1, h(1.9) = 4.85$$



- 11 Find the domain of the function and identify any vertical and horizontal asymptotes. *Lose asymptote!*

$$f(x) = \frac{x-4}{x^2 - 16} = \frac{x-4}{(x-4)(x+4)} = \frac{1}{x+4}$$

Domain: \mathbb{R} except $x = \pm 4$

Vertical Asymptote(s): $x = \pm 4$

Horizontal Asymptote(s): $y = 0$

- 12 Solve the inequality and state the solution using interval notation.

$$\frac{x+6}{x+1} - 2 < 0$$

$$\frac{x+6 - 2(x+1)}{x+1} < 0$$

$$\frac{-x+4}{x+1} < 0$$

<u>Num.</u>	<u>Den.</u>
$-x+4=0$	$x+1=0$
$x=4$	$x=-1$

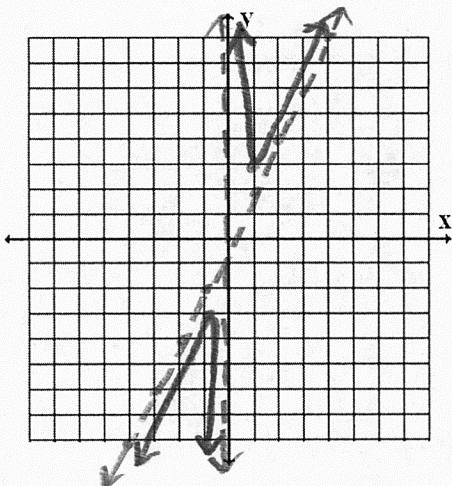
Test:



$$(-\infty, -1) \cup (4, \infty)$$

13 Graph.

$$f(x) = \frac{2x^2 + 1}{x}$$



Vertical Asymptote: $x = 0$

Slant Asymptote: $y = 2x$

$$\begin{array}{r} x \\ \hline -1 & | & -3 \\ 1 & | & 3 \\ \hline 2x+0 & & \\ \times [2x^2+0x+1] & & \\ \hline -2x^2 & & \\ \hline 0x+1 & & \\ -0x-0 & & \\ \hline 1 & & \end{array}$$

$$2x + \frac{1}{x}$$

14 Path of a Diver The path of a diver is given by

$$y = -2x^2 + 12x - 6$$

where y is the height (in feet) and x is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver?

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = \frac{-12}{-4} = 3$$

$$y = -2(3)^2 + 12(3) - 6$$

$$y = \boxed{12 \text{ ft}}$$

15 Solve the inequality and state the solution using interval notation.

$$x^3 - 2x^2 - 9x - 2 \geq -20$$

+20 +20

$$x^3 - 2x^2 - 9x + 18 \geq 0$$

$$x^2(x-2) - 9(x-2) \geq 0$$

$$(x-2)(x^2-9) \geq 0$$

$$(x-2)(x+3)(x-3) \geq 0$$

Critical #'s: $x=2$ $x=-3$ $x=3$

$$\begin{array}{ccccccc} & & & & & & \text{Test} \\ & - & & + & & - & + \\ \leftarrow & & -3 & & 2 & & 3 \rightarrow \\ & & & & & & \end{array}$$

$$\boxed{[-3, 2] \cup [3, \infty)}$$

16 Use the information given to state the complete factorization of the polynomial and its solutions (zeros).

Polynomial Equation	Value of x
$x^3 - 7x + 6 = 0$	$x = 2 \rightarrow x-2 = 0$

Factorization: $(x-2)(x-1)(x+3)$

Solutions: $x=2, x=1, x=-3$

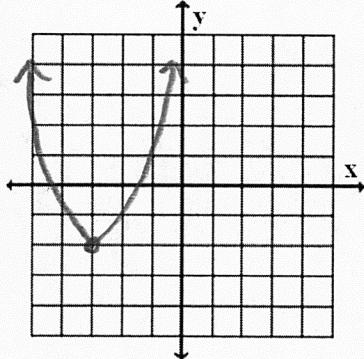
$$\begin{array}{r} 2 | 1 & 0 & -7 & 6 \\ \downarrow & 2 & 4 & -6 \\ 1 & 2 & -3 & 0 \end{array}$$

$$x^2 + 2x - 3$$

$$(x-1)(x+3)$$

- 17 Sketch the graph of the function.

$$f(x) = (x + 3)^2 - 2 \quad \text{Vertex: } (-3, -2)$$



- 18 Use synthetic division to divide.

$$(3x^3 - 17x^2 + 15x - 25) \div (x - 5)$$

$$\begin{array}{r} 5 \\ | \quad 3 \quad -17 \quad 15 \quad -25 \\ \downarrow \quad 15 \quad -10 \quad 25 \\ \hline 3 \quad -2 \quad 5 \quad 0 \end{array}$$

$$\boxed{3x^2 - 2x + 5}$$

- 19 Write the quotient in standard form, $a+bi$.

$$\frac{2}{4-5i} \cdot \frac{(4+5i)}{(4+5i)}$$

$$\frac{8+10i}{16-25i^2} = \frac{8+10i}{16+25}$$

$$= \frac{8+10i}{41}$$

$$= \boxed{\frac{8}{41} + \frac{10}{41}i}$$

- 20 Re-write the function in standard form. State the vertex, axis of symmetry, and x-intercepts for the quadratic function.

$$h(x) = 4x^2 - 4x + 21$$

$$\text{Standard Form: } h(x) = 4\left(x - \frac{1}{2}\right)^2 + 20$$

$$\text{Vertex: } \left(\frac{1}{2}, 20\right)$$

$$\text{Axis of Symmetry: } x = \frac{1}{2}$$

x-intercept(s): none

$$h(x) = 4(x^2 - x) + 21$$

$$\text{x-intercept: } 4(x^2 - x + \frac{1}{4}) + 21 - \frac{1}{4}(4)$$

$$\rightarrow 0 = 4\left(x - \frac{1}{2}\right)^2 + 20 \leftarrow \text{Standard Form}$$

$$\pm \sqrt{-20} = \sqrt{4\left(x - \frac{1}{2}\right)^2}$$

\uparrow
Not real!