## Pg. 236 3.2 - Logarithmic Functions and Their Graphs

The inverse of the exponential function $\mathrm{a}^{\mathrm{x}}$ is called the logarithmic function with base a.

$$
\log _{a} x \text { is read as "log of } x \text { base a." }
$$

Logarithmic and exponential equations are interchangeable.

## Logarithmic Form

## Exponential Form

$$
\mathrm{y}=\log _{\mathrm{a}} \mathrm{x} \longleftrightarrow \mathrm{x}=\mathrm{a}^{\mathrm{y}}
$$

Important Note: Bases are the same both in logarithmic and exponential form.
You can NOT take the log of a negative number.
The most common logarithm has base 10. If there is no base for the logarithm, then it is 10 .

## Ex 1:

Convert to exponential form.
a) $\log _{3} 81=4$
b) $\log _{10} \frac{1}{1,000}=-3$

## Ex 2:

Solve without a calculator.
Solve with a calculator.
a) $f(x)=\log _{16} x$ for $x=4$
b) $f(x)=\log x$ for $x=\frac{1}{500}$

## Ex 3:

Convert to logarithmic form.
a) $8^{2}=64$
b) $9^{3 / 2}=27$

Natural Logarithm: $\ln x=\log _{e} x$

## Ex 4:

Convert to exponential form.
Convert to logarithmic form.
a) $\ln \frac{2}{5}=-0.916 \ldots$
b) $\mathrm{e}^{2}=7.3890 \ldots$

## Properties of Logarithms

$\log _{\mathrm{a}} 1=0 \quad$ because $\mathrm{a}^{0}=1$
$\log _{\mathrm{a}} \mathrm{a}=1 \quad$ because $\mathrm{a}^{1}=\mathrm{a}$
$\log _{a} a^{x}=x \quad$ and $a^{\log _{a} x}=x$
If $\log _{a} x=\log _{a} y$, then $x=y$

Properties of Natural Logarithms
$\ln 1=0 \quad$ because $\mathrm{e}^{0}=1$
$\ln \mathrm{e}=1 \quad$ because $\mathrm{e}^{1}=\mathrm{e}$
$\ln e^{x}=x \quad$ and $e^{\ln x}=x$
If $\ln x=\ln y$, then $x=y$

## Ex 5:

Simplify. Note: May help to set expression equal to x .
a) $\log _{1.5} 1$
b) $9^{\log _{9} 5}$

Ex 6:
Evaluate with a calculator.
Solve without a calculator.
a) $f(x)=3 \ln x$ for $x=0.32$
b) $f(x)=\ln x$ for $x=e^{-2}$
c) $\log _{2}(\mathrm{x}-3)=\log _{2} 9$

The inverse function of an exponential function with base a is a logarithmic function with base a.

## Graph of Logarithm



Graph of Natural Logarithm


## Parent Logarithmic Function

Domain: $(0, \infty)$
x-intercept (Starting Point): (1,0)

## Vertical Asymptote: $x=0$

## Ex 7:

Find the domain, $x$-intercept, and vertical asymptote of the logarithmic function and sketch its graph.
a) $g(x)=\log _{6} x$
b) $h(x)=\ln (x+1)$


Domain:

Assignment 3.2
Vertical Asymptote:


Pg. 236 Vocab \#'s 1-5

## Problem Set \#'s 1-85 ODD

